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ON THE THERMAL CONDUCTIVITY OF CAST IRON.

By E. H. Hall and C. H. Ayres.

With Two Plates.

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ON THE THERMAL CONDUCTIVITY OF CAST IRON.

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Two or three years ago an article entitled "On the Thermal Conductivity of Mild Steel" was published by one of the authors * of the present paper. The method described at length in that article made use of a disk of soft steel, about 0.3 cm. thick and about 10 cm. in diameter, coated on each face with an electrolytic deposit of copper about 0.05 cm. thick. Thin copper wires attached electrolytically to these copper coatings led to a sensitive galvanometer, the deflections of which depended upon the thermo-electromotive force of the couple made by the steel of the disk and the copper of its coverings, and indicated the difference of temperature existing between the two faces of the steel itself.

Water of a known temperature was made to flow across one copper face of the disk and water eight or ten degrees warmer across the other copper face. The water delivery of one stream was measured, and its change of temperature between entering and leaving the vessel containing the disk was determined by means of two copper and German silver thermo-electric junctions.

The apparatus containing the disk was surrounded by a water jacket having a temperature near that of the disk, so that the radiation to or from the exposed convex surface of the disk could be neglected.

This hasty review shows that, if all the measurements indicated were correctly made, the thermal conductivity could be found by a simple calculation based on the data afforded by the experiments. In fact, the experiments described in the article under discussion left something to be desired; for they showed that different parts of the same face of the disk were not at the same temperature, and the process of calculation necessary to deduce from the observations the mean difference of temperature of the two faces of the disk was laborious, and perhaps to the casual reader not entirely convincing. This difficulty and the desirability of certain changes in the apparatus used were recognized in the article itself.

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* E. H. Hall, These Proceedings, Vol. XXXI. p. 271, 1896.

The authors of the present paper have used the same method, applying it to measure the thermal conductivity of a certain grade of cast iron; but they have employed a thicker disk and thicker layers of copper. The result of these changes has been to produce such uniformity of temperature over each face of the disk, that calculation of the mean difference of temperature between the two faces of the disk has become an exceedingly simple matter.

The Iron Used.

The disk was made from a slab of cast iron the origin and description of which are well set forth in the following extract from a letter written by Mr. A. C. Colby, the metallurgical engineer of the Bethlehem Iron Company: —

"Dear Sir, — In response to the instructions contained in your letter of the 30th ultimo, I send to-day . . . the casting which has been made at these works, and for which no charge will be made to you. I selected a high silicon iron so as to make the casting free from any chill, and it is smooth as can be obtained in a sand lined mould, and, I think, very close to the dimensions you desire, namely, 12″ × 4″ × 1″.

"In the following composition of the iron entering into this casting, the sulphur and silicon determinations were made on a gate of the casting. The other determinations are approximate, based on our daily analyses from the furnace from which this casting was made: —

Carbon	3.40 – 3.60
Manganese	.50 – .55
Phosphorus	.053 – .058
Copper	.050 – .055
Sulphur	.106
Silicon	1.40."

Dimensions and Treatment of Disk.

The diameter of the disk made from this casting was 10.06 cm.; its mean thickness was called 1.787 cm.; the largest of nine measurements at different places indicating 1.798 cm., and the smallest 1.776 cm.

The copper plating of the disk was effected by giving it first a thin coating from a cyanide of copper solution, such as is used by nickel-platers in preparing iron to receive the nickel, and then finishing the operation by use of a sulphate of copper solution. Much preliminary experimenting in this process was done on a block of the cast iron before the disk was taken in hand, in order to make sure that a good

and strongly adherent coating would be obtained upon the latter at the first trial. The final procedure, which worked well, was as follows: —

Two electrolytic baths were prepared, one consisting of a cyanide of copper solution purchased ready made from a nickel plater, the other being an ordinary sulphate of copper solution of specific gravity about 1.10, acidulated by about one drop of strong sulphuric acid to ten cubic centimeters of the solution. Each solution contained two vertical plates of copper, somewhat broader than the disk to be coated, placed several centimeters apart.

A hole about 0.5 cm. in diameter was bored a short distance into the curved side of the disk, and in this was fixed one end of a steel rod, which was to serve the double purpose of a handle and a conductor of the current from the disk. After being rubbed tolerably bright the disk was boiled in a strong solution of caustic potash for ten minutes, then rinsed in flowing water, then scoured with powdered pumice and water by means of a bristle brush, then dipped some seconds in a 20% solution of hydrochloric acid, then rinsed again in flowing water, then dipped again in the acid solution, then rinsed again, then placed between the two plates of copper in the cyanide of copper solution, which was at a temperature near 70° C., and kept there half an hour with a current of about 3.5 amperes flowing through it. At the end of this time the surface of the disk, including its curved side, was well coated with copper. Accordingly, the disk was taken from the solution, rinsed, covered as to its curved surface with a rubber band to prevent further deposit of copper there, then placed in the sulphate of copper solution between two copper plates about 8 cm. apart; and a current of 3 amperes or more was set to flow through it.

After a number of hours, beads of copper were seen to have formed at the edges of the two flat faces of the disk, and the disk was removed from the solution long enough to allow these beads to be broken or filed off. It was then rinsed, dipped in the hydrochloric acid solution, rinsed again, then replaced in the sulphate bath and again subjected to the current. This course of operations was continued for several days, about 135 hours of current use, until the layer of copper on each face of the disk was about 0.2 cm. thick. At one stage of the procedure it was found necessary to resort to the cyanide bath again for a short time, the filing off of the copper beads at the edges having exposed the iron at certain parts of the convex surface.

The coatings when completed were somewhat thicker near the edge than in the middle. Accordingly, they were turned off in the lathe to a

nearly uniform thickness, though one of them was left slightly thicker at the edge. The final thickness of each coating was not far from 0.2 cm. The convex surface of the disk was turned off sufficiently to leave a good surface and show clearly the junctions of the copper coatings with the iron body. The diameter of the disk was thus reduced to 9.94 cm.

Each coating was now channelled at the edge to a depth of about 0.1 cm. and a width of about 0.17 cm., as in Figure 1 of Plate I., and a brass ring, R or R', 0.3 cm. thick, was shrunk into the channel in each coating. R' is cut through in Figure 1. The object of this detail will appear presently.

Mounting and Use of the Disk.

Figure 2 shows how the disk was placed and used in the experiments on conductivity. In this figure, the scale of which is $\frac{1}{2}$, I represents the disk; c and c' are the copper coatings; and the rings just described can be seen set into the edges of the coatings. The lower ring is shown cut through by horizontal passages. There are, in fact, in this ring 24 horizontal slots, each about 1 cm. long and 0.2 cm. wide, the object of which is to allow the water entering vertically beneath the middle of the disk to flow out horizontally from beneath the disk, thus carrying away the air-bubbles which warm water inevitably contains, and which would accumulate beneath the disk if an immediate downward escape of the water through small passages were required. Upon passing from beneath the disk the water enters a groove cut in a hard rubber ring, $h'h'$, and covered by a brass flanged ring $n\,n$ fastened to $h'h'$. Thence it passes downward and out of the apparatus by several passages of 4 or 5 millimeters in diameter, only two of which are shown in the figure. The slotted brass ring through which the water flows carries the iron disk, and rests in a groove in the hard rubber ring $h'h'$, a soft rubber tube at the bottom of this groove making a water-tight packing. The ring $h'h'$ has at the bottom another groove, which receives the top of the brass ring $r'r'$, which rests upon a wooden support to which it is firmly attached by means of a horizontal flange. Soldered within $r'r'$ near its top is the brass plate $p'p'$, which carries the hard rubber block $H'H'$, in the centre of which is fixed the tube that carries the water to the bottom of the disk. This tube is enclosed for a part of its length by another, which extends downward from $p'p'$; but this is an unimportant detail.

Encircling the iron disk is a soft rubber band $b\,b$, which was intended partly as a protection of the iron against rusting, and partly as a dam to prevent leakage of water upward past $n\,n$. Another similar band, not

shown, rested its lower edge upon $n n$; but such precaution against leakage was perhaps hardly necessary. The downward escape of water from the groove in $h'h'$ was so free that there was little tendency for it to overflow $n n$.

Starting again at the iron disk and now proceeding upward, we find $h h$, $H H$, and $p p$, corresponding in material and in general shape and position to $h'h'$, $H'H'$, and $p'p'$, already mentioned. The mere weight of the apparatus being insufficient to prevent vertical movement and dislocation under the pressure of the water within, a retaining device was used, which is described as follows. A flat ring of brass, not shown in Figure 2, was provided with three internal radial offsets, each of which bore upon a block of wood resting upon the narrow external flange of $p p$. Three brass bolts led from this ring to the brass base-plate of the apparatus, enabling the experimenter to apply to the plate $p p$ any necessary amount of downward pressure.

Certain other parts in the upper portion of the figure require explanation. The parts there shown in dotted outline do not lie in the median section of the apparatus, and are to be regarded as behind the plane of the rest of the figure. For example, the vertical tube indicated above J_1 does not rise directly from J_1, but from a horizontal offset extending from J_1 as in Figure 3. Another horizontal offset from J_1 receives, as the same figure shows, one end of a plug, P_1, consisting of two semi-cylindrical pieces of hard rubber pressing between them a strip of soft rubber packing, which packing separates the wires of the copper-German silver junction j_1. J_2, Figure 2, is similar to J_1, and contains a similar junction. More will be said of these junctions later.

Water entering at A flows vertically past the bulb of the thermometer T_1, which gives a rough indication of its temperature, then horizontally past the junction in J_1, then by a brass tube into the funnel-shaped passage FF, then downward through numerous holes near the edge of $p p$, and so on, as the arrows show, under HH, upward through the vertical brass tube t_1, which touches the enclosing brass tube t_2 only near the ends of the latter, past the other copper-German silver junction within J_2, past the bulb of T_2, thence out by means of a rubber tube to the lower part of the jacket KK, around and upward through this jacket to the main outlet at O. The jacket has a supplementary outlet at S; and the water from both outlets is collected and weighed below.

Leading upward from the apex of the funnel FF is a small tube, w, through which a slight waste flow of water is maintained in order to carry off air-bubbles from FF. Two openings in the top of the water

jacket give escape for air, and allow the use of thermometers for taking the temperature of the water in the jacket.

This water flows over as well as around the enclosed apparatus. The opening in the double top of the jacket, through which extend the tubes shown above J_1 and J_2 in Figure 2, is about 7.5 cm. by 2.5 cm. Below J_1 and J_2, down to the hard rubber ring $h\,h$, the tubes and funnel were thickly wrapped with cotton to lessen radiation between these parts and the jacket. The space between $h\,h$ and $h'h'$, as well as that around and below $h'h'$, was carefully and fully packed with the same material.

Figure 2 shows two fine copper wires leading out from the coating C, and passing through holes in the hard rubber ring $h\,h$, where they are held in place by means of hard rubber plugs, k_1 and k_2, with soft rubber packing. There are, in fact, see Figure A, p. 290, thirteen such wires, 0.018 cm. in diameter, attached to C by electrolytic deposit of copper by a process sufficiently described in the article referred to in the beginning of this paper. Wire no. 13 is attached at the centre of C; nos. 3, 6, 9, and 12 are attached symmetrically about 2 cm. from the centre; nos. 2, 5, 8, and 11, symmetrically about 3.2 cm. from the centre; nos. 1, 4, 7, 10, symmetrically about 4.4 cm. from the centre. Similar wires, nos. 1′–13′, are similarly attached to the coating C', no. 1′ being immediately beneath no. 1, no. 2′ immediately beneath no. 2, etc. These wires pass through the ring $h'h'$ exactly as nos. 1–13 pass through the ring $h\,h$. To prevent deposit of copper upon the free parts of the wires during the process of attachment, and to prevent illegitimate metallic contacts between the wires and the coatings C and C' during the experiments on conduction, the wires were coated with shellac between the points of attachment to the coatings and the places of exit through the hard rubber rings. Outside the rings each wire was led to a point on a wooden platform, where it was, by means of a screw and copper washers, held in firm copper connection with a larger copper wire. The twenty-six larger wires thus brought into connection led to a like number of small mercury wells in a board placed at some distance from the apparatus shown in Figure 2.

Determination of the Difference of Temperature of the Two Faces of the Disk.

The mercury wells were so arranged that by means of copper connectors reaching from one well to another any point of junction on the upper coating of the iron disk could be thrown into circuit with the corresponding point on the under coating and with an astatic galvanometer.

Care was taken to make the thirteen circuits which could be, one at a time, thus formed very nearly equal in resistance. It was possible to use such copper connectors between the mercury wells as to throw all the wires leading from the upper coating of the disk into multiple arc with each other, and all those leading from the lower coating into multiple arc with each other, and to connect the two sets of wires in one circuit with the galvanometer. The latter mode of connection was finally used in the conductivity experiments; but certain preliminary observations with the single circuits were made in order to find whether the various pairs of junctions on the disk were enough alike in performance to justify connecting them in multiple. The method of testing was to run a stream of water at constant temperature through the apparatus on the under side of the disk, and another stream at a different constant temperature through on the upper side of the disk, and to note the galvanometer deflections obtained from each of the pairs of junctions in turn. The following table shows the result of the observations:—

Deflections.

Junctions.	Oct. 23.	Oct. 26.	Oct. 26.	Nov 3.	Nov. 8.	Mean.	
13 and 13′	4.91	4.38	4.46	4.86	4.74	4.67	
1 and 1′	5.31	4.90	4.87	5.25	5.19	5.10	4.83
4 and 4′	5.08	4.78	4.70	5.20	5.00	4.95	
7 and 7′	4.93	4.55	4.56	4.80	4.60	4.69	
10 and 10′	4.71	4.53	4.40	4.75	4.55	4.59	
2 and 2′	5.06	4.70	4.51	5.15	5.05	4.89	4.73
5 and 5′	5.03	4.70	4.63	5.06	5.00	4.88	
8 and 8′	4.76	4.47	4.40	4.75	4.68	4.61	
11 and 11′	4.66	4.33	4.30	4.76	4.68	4.55	
3 and 3′	4.90	4.67	4.59	5.02	4.98	4.83	4.68
6 and 6′	4.70	4.67	4.59	4.99	4.98	4.79	
9 and 9′	4.66	4.40	4.40	4.76	4.70	4.58	
12 and 12′	4.60	4.33	4.35	4.65	4.70	4.53	

An examination of this table, in connection with Figure A, leads to the conclusion that the mean difference of temperature between the two sides of the disk increases from the centre to the circumference about 3 per cent. It appears, too, that the mean difference of temperature between top and bottom is greater along the radius 1–2–3–13 than along the

radius 4–5–6–13, greater along the latter than along 7–8–9–13, greater along the last than along 10–11–12–13. The mean difference of temperature along 1–13 apparently exceeded that along 10–13 about 6 per

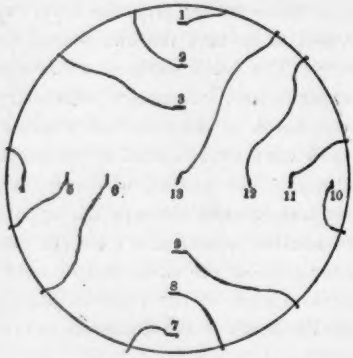


FIGURE A.

cent. A partial explanation of this difference probably is that, on one face or the other of the disk, the flow of water was freer in the region 1–13–4 than in the region 7–13–10. This inequality of flow might be caused by a slight tilting of *HH* or of *H'H'* (Fig. 2).

Two short sets of observations were made to compare the deflections given by all the pairs of junctions in multiple with the mean of the deflections given by the pairs used separately. The pair 5–5′ was now found defective, and was omitted from the comparison. The results, after allowance for the smaller resistance of the multiple arc, were as follows: —

	From Multiple Arc.	From Single Pairs.	Ratio.
Nov. 16	4.47	4.37	1.023
" 20	4.37	4.31	1.014

The test of November 20 was the more careful of the two; but even in this test the difference between the ratio found, 1.014, and unity, was within the possible limits of error of observation.

The tests which have been described were considered to justify using the junctions in multiple and treating the current obtained from the combination as representing the mean thermo-electromotive force of the

whole disk, and therefore the mean difference of temperature of the two faces of the iron. There is doubtless some inaccuracy in this conclusion. Strictly, somewhat greater weight should be given to the indications from the outer circle of junctions than to those from junctions nearer the centre; for an inspection of Figure A will show that each of the outer junctions represents a somewhat greater area than one of the inner junctions. The multiple arc method of operation makes no allowance for this fact, but the error from this cause was probably very small. It is to be observed, moreover, that an error of 1%, for example, in the absolute value of the thermal conductivity of a particular piece of iron is of no great consequence in the present state of investigation, provided the change of conductivity with change of temperature can be determined with some degree of accuracy.

Before the experiments upon conductivity were made, a number of the fine wires leading from the faces of the disk having failed, a complete new set of wires, from the same piece as the first set, was put in by the same method and in the same positions as before. The apparatus was then set up once more, in its former condition as nearly as possible.

The strength of the electric current coming from the disk was measured by means of an astatic galvanometer, the sensitiveness, or figure of merit, of which was determined frequently by sending through it a known fraction of a current measured by a good tangent galvanometer. The resistance of the circuit containing the disk and the galvanometer being known, the thermo-electromotive force producing the current from the disk was found. But before this e. m. f. could be translated into difference of temperature between faces of the iron, it was necessary to determine by experiment the e. m. f. arising from some known difference of temperature between two junctions made of copper and of iron like the iron of the disk. For this test a piece about 10 cm. long and 0.16 cm. in diameter was cut from the same slab of cast iron from which the disk had been taken; and to each end of this slender bar a copper wire, from the same piece as the wires attached to the coatings of the disk, was fixed by electrolytic deposit of copper. The bar was set in a hard rubber holder, about 2.5 cm. projecting at each end, and the whole was mounted between two brass tubes in such a way (see Fig. 4) that water flowing through either tube would flow over one end of the bar. Thus water entering at A_1 ran past the bulb of the thermometer T_1 along the end I_1 of the iron, and out at E_1. An alternative exit for the water is indicated by the dotted lines below I_1. A coating of shellac was used to protect the iron and the copper from the chemical action of the water.

In order to eliminate various sources of possible error, including especially disagreement of the two thermometers, sets of observations were made in pairs, the stream entering at A_1 being warmer than the other in the first set of observations and colder than the other in the following set. The difference of temperature was usually between 5° and 10° C.

Some doubt was felt at first as to whether the ends of the bar would have the same difference of temperature as the thermometers. It is evident that use of exits E_1 and E_2, whereby the streams were made to flow a considerable distance along the bar, would be more effective than the use of the dotted exits. The latter were used upon occasion, with the idea that, if they gave about the same effect as E_1 and E_2, the latter could be regarded as satisfactory. The dotted exits gave a result some four or five per cent less than that given by the other exits. It appeared unlikely that any considerable error would be made in assuming that, when E_1 and E_2 were used, the difference of temperature of the ends of the bar was the same as the difference of temperature of the thermometers.

It will be observed, however, that the iron bar used in this test was like a piece cut parallel to a certain *diameter* of the disk, not parallel to the thickness of the disk. It was a matter of very grave doubt whether the thermo-electric quality of the bar, taken lengthwise, could be regarded as identical with the thermo-electric quality of the disk taken thickness-wise. It was the latter quality that came into play in the conductivity experiments; and some way of determining it was to be found. The method which was finally adopted is described in Appendix I. of this paper. It showed that scarcely an appreciable error would have been made by using the results obtained from the first method, above described.

With the information thus obtained it was easy to calculate with considerable accuracy the difference in temperature of the two faces of the iron disk in the conductivity experiments, the indications of the astatic galvanometer being readily interpreted. The deflections of the galvanometer, upon reversal of the current, were usually about 9 cm.; and the difference of temperature of the faces of the iron was usually rather more than 1° C. The difference of temperature of the streams, above and below the disk, was usually about 8° C.

Determination of the Difference of Temperature of the Ingoing and Outcoming Water at the Chamber above the Disk.

The method of making this measurement has already been indicated. Two copper-German silver junctions, each like that shown in Figure 3, were used, one at J_1 in Figure 2, the other at J_2 in the same figure. The German silver wire used, about 0.015 cm. in diameter, was continuous from J_1, Figure 3, to the corresponding junction in the other plug. Its length was perhaps 30 cm. The fine copper wire, 0.018 cm. in diameter (from the same piece as the wires attached to the coatings of the disk), leading from J_1, Figure 3, did not extend completely through the hard rubber plug, or holder, but was soldered carefully, some distance from the outer end of the plug, to a larger copper wire, which led off toward an astatic galvanometer. The arrangement of copper wires at the other plug was quite similar. The fine wires of each junction were coated thinly with shellac.

The two copper-German silver junctions thus described, or similar ones,* were tested, or "calibrated," by means of streams of water, of a known difference of temperature, flowing past the junctions according to the arrows in Figure 3. The difference of temperature of the streams was found by means of the same pair of thermometers that are indicated in

* In accordance with my advice, Mr. Ayres made only such experiments in the calibration of his copper-German silver junctions as to show that their performance differed but little from that of similar junctions used previously by myself. After this, in all his calculations of the conductivity, he took his values of the thermo e. m. f. of copper-German silver from Figure 8 of my previous paper, already referred to, "On the Conductivity of Mild Steel." In preparing the present paper, I have had some misgivings as to the accuracy of these values, and therefore in October, 1898, I made more experiments upon a pair of junctions quite similar to those used by Mr. Ayres. The results are given in the second column below. The third column gives values, for the same temperatures, taken from the figure used by Mr. Ayres: —

Mean Temp.	Electromotive Force, in Volts, per 1° C. Difference of Temperature of Junctions.	
20°.3	.00001743	.00001732
37°.9	.00001826	.00001830
58°.2	.00001944	.00001942

From the old observations and the new combined a curve representing the thermo e. m. f. at temperatures ranging from 15° to 65° was constructed, and the values of the conductivity found by Mr. Ayres were revised accordingly. The resulting changes of conductivity were slight, but they had considerable effect upon the estimated temperature coefficient of conductivity. — E. H. H.

Figure 4, and the same method of alternating the hotter and cooler streams was used here that was used in the test of the copper-iron junctions. The difference of temperature of the streams in the calibration tests was usually about 4° or 5° C.

In the conductivity experiments proper, the usual difference of temperature of the copper-German silver junctions, the usual difference of temperature, that is, of the ingoing and outgoing water, was probably rather more than 0°.5 C.

The Flow of Water.

The method of controlling and heating the streams of water was essentially the same as that described in the previous paper,* and illustrated in Figure 5 of that paper. Powerful gas-burners, of a type manufactured by the Buffalo Dental Company and expressly intended for heating streams of water, were used. Each stream flowed through the conduction apparatus from the base of an overflowing standpipe, which device answered the double purpose of insuring a constant flow, and allowing air bubbles to escape from the water before reaching points where they would do harm. A supplementary air-vent was provided for the upper stream near its entrance at *A*, Figure 2.

The stream which flowed above the disk, the only one upon which careful measurements were made, ran into a covered barrel standing upon a platform balance. The time of flow was noted, and the amount of water accumulating in the barrel during that time was determined by weighing. The rate of delivery of the stream ranged, during the whole course of the investigation, from about 15 grams per second to about 25 grams per second. The stream flowing beneath the disk was of the same order of magnitude.

Results and Discussion.

A few sets of observations were made at low temperatures without the use of the jacket. A few others were made at various temperatures with use of jacket, but without the cotton packing within and below it. These observations were preliminary, and none of them will be used in deducing the final results. The detailed results of subsequent observations, made with jacket and cotton packing in use, are given below in chronological order. None are omitted, although some are placed in brackets for reasons to be given later.

* These Proceedings, Vol. XXXI., 1896.

The "mean temperature" is the mean between the temperature of the upper stream upon entering the apparatus (as indicated by the thermometer T_1 in Fig. 2), and the lower stream upon leaving * the apparatus. Neither of these temperatures was taken with great accuracy, and any one of the mean temperatures given may be wrong to the extent of 0°.5.

Date, 1897.	Mean Temp.	K_1	K_2	K
May 11	21°	0.1471	0.1495	0.1483
" 13	19°.6	0.1489	0.1558	0.1524
" 15	20°.4	0.1503	0.1511	0.1507
" 18	39°.1	0.1485	0.1489	0.1487
" 20	40°.9	0.1512	0.1520	0.1516
" 25	22°	0.1522	0.1515	0.1519
" 26	40°.2	0.1482	0.1494	0.1488
" 28	20°.6	0.1533	0.1507	0.1520
" 29	35°.7	0.1523	0.1494	0.1509
June 7	74°	0.1421	0.1309	0.1365
" 8	72°.9	0.1523	0.1559	0.1541
" 21	77°.3	0.1382	0.1400	0.1391
" 23	21°.7	0.1536	0.1465	0.1501
" 26	56°.2	0.1506	0.1407	0.1457
[July 1	55°.5	0.1741	0.1443	0.1592]
[" 2	61°.6	0.1584	0.1188	0.1386]
[" 8	58°.5	0.1564	0.1425	0.1495]
" 23	54°.2	0.1518	0.1487	0.1503
" 24	57°.3	0.1485	0.1423	0.1454
" 26	28°.4	0.1496	0.1504	0.1500
[" 30	74°.6	0.1368	0.1588	0.1478]
Aug. 3	56°.5	0.1427	0.1441	0.1434
" 4	27°.5	0.1557	0.1528	0.1543
" 7	27°.0	0.1519	0.1515	0.1517
" 8	59°.2	0.1470	0.1446	0.1458

Under K_1 are given values of the conductivity obtained from observations made when the warmer stream ran above the disk. Under K_2 are given values of the conductivity obtained from observations made when the warmer stream ran below the disk. K is the mean of K_1 and K_2. In the calculation of the values here given, no account was taken of the

* The temperature of the lower stream on *entering* the apparatus was not taken; but the change of temperature within the apparatus was slight.

variation of the specific heat of water with variation of temperature, this specific heat being called 1 for every temperature used. This inaccuracy will be referred to again.

These figures show a considerable range of temperature, and from them it should be possible to derive an approximate value at least of the temperature coefficient of K. The numbers given in brackets, however, will not be used for this purpose. The numbers for July 1, 2, and 3 exhibit great differences between K_1 and K_2, and also between the values of K. On those days, and those only, the pair of copper-German silver junctions, used to determine the change of temperature of the upper stream, were covered with shellac *melted* on. The coating thus obtained was too thick, so that the junctions did not take the temperature of the water with sufficient readiness. There are in the table above given other values of K obtained at temperatures not very different from those at which these rejected values were found. The values obtained for K at all temperatures above 70° differ much among themselves; but it hardly seems best to reject them altogether in the attempt to arrive at an approximate value of the temperature coefficient of K. The great variation observed among them was probably due to unsteadiness of temperature of the water streams when very hot, or to possible impairment by the hot water of the shellac coating on the copper-German silver junctions.

All the values of K not contained within the bracketed lines will be used in some fashion in estimating the temperature coefficient; but they will be used in two divisions, one for May and June, the other for July and August. The reason for this division is that on August 5 some of the lines of wire leading from the copper coatings on the iron disk were found to be out of condition. The pairs of wires affected were 1, 2, 11, and 12, the other nine pairs remaining in good condition. When this partial breakdown began it is impossible to determine; June 30 was the last date on which all the pairs of wires were known to be in good order. It has been shown in the early part of this paper that each pair of wires gave about the same effect as any other pair; therefore, as all were joined in multiple, the failure of a few of them should affect the total current but little, the resistance of the remaining pairs being but a small part of the total resistance of the circuit. The failing pairs lay, one in the outermost circle, two in the next, and one in the next. It appears, from a comparison of the values of K obtained near 21° and near 39° in May and June with the values obtained near 28° in July and August, that the impairment of the wires or some other unknown

cause made the later values at a given temperature about one per cent greater than they would have been had they been obtained at the same temperature in May or June. In these earlier months sets of observations were made at various temperatures from near 20° to near 75°. In July and August sets near 28° were intermingled with sets near 57°. It is possible, therefore, to make for each period an independent determination of the temperature coefficient of K.

We have from the May and June division: —

Date.	Mean Temp.	K_1	K_2	K
May 11	21°	0.1471	0.1495	0.1483
" 13	19°.6	0.1489	0.1558	0.1524
" 15	20°.4	0.1503	0.1511	0.1507
" 25	22°	0.1522	0.1515	0.1519
" 28	20°.6	0.1533	0.1507	0.1520
June 23	21°.7	0.1536	0.1465	0.1501
	20°.9	0.1509	0.1509	0.1509
May 18	39°.1	0.1485	0.1489	0.1487
" 20	40°.9	0.1512	0.1520	0.1516
" 26	40°.2	0.1482	0.1494	0.1488
" 29	35°.7	0.1523	0.1494	0.1509
	38°.9	0.1501	0.1499	0.1500
June 26	56°.2	0.1506	0.1407	0.1457
June 7	74°	0.1421	0.1309	0.1365
" 8	72°.9	0.1523	0.1559	0.1541
" 21	77°.3	0.1382	0.1400	0.1391
	74°.7	0.1442	0.1423	0.1432

From July and August we have: —

Date.	Mean Temp.	K_1	K_2	Mean K
July 26	28°.4	0.1496	0.1504	0.1500
Aug. 4	27°.5	0.1557	0.1528	0.1543
" 7	27°.0	0.1519	0.1515	0.1517
	27°.6	0.1524	0.1516	0.1520
July 23	54°.2	0.1518	0.1487	0.1503
" 24	57°.3	0.1485	0.1423	0.1454
Aug. 3	56°.5	0.1427	0.1441	0.1434
" 8	59°.2	0.1470	0.1446	0.1458
	56°.8	0.1475	0.1449	0.1462

The single set of observations, made July 30, at a temperature near 75° is hardly worth taking into account here, the uncertainty of observations at such a temperature being great, as we have seen.

According to the evidence thus far we have, from the May and June observations,

at 20°.9	$K = 0.1509$
" 38°.9	" $= 0.1500$
" 56°.2	" $= 0.1457$ (?)
" 74°.7	" 0.1432 (?)

and from the July and August observations,

at 27°.5	$K = 0.1520$
" 56°.8	" $= 0.1462$

As the change of K with change of temperature appears to be small in any case, it becomes important to consider the change of specific heat of water with change of temperature; for all values which precede are given on the assumption that the specific heat of water is 1 at all temperatures used.

Winkelmann, in Part II. of Volume II., p. 340, gives a table of the specific heats of water, which he has deduced from a formula proposed by himself after a discussion of the results obtained by numerous experimenters. This table gives: —

Temp.	Sp. Heat.	Temp.	Sp. Heat.
0° C.	1.0000	50° C.	0.9939
10°	0.9944	60°	0.9992
15°	0.9924	70°	1.0067
20°	0.9910	80°	1.0164
25°	0.9901	90°	1.0283
30°	0.9898	100°	1.0424
40°	0.9907		

Revising, in accordance with this table, the values of K last given above, we get,

at 20°.9	$K = 0.1494$
" 38°.9	" $= 0.1485$
" 56°.2	" $= 0.1453$ (?)
" 74°.7	" $= 0.1447$ (?)
" 27°.6	" $= 0.1505$
" 56°.8	" $= 0.1458$

The difference between the value of K found for 20°.9 and that found for 38°.9 is so slight that very little importance can be attached to it, in view of the much greater differences between successive measurements of K at or near any one temperature. Making the formal calculation, however, from these values as they stand, we get, as the temperature coefficient of K between 20°.9 and 38°.9,

$$\frac{0.1494 - 0.1485}{0.1494\,(38.9 - 20.9)} = -0.00033.$$

Taking the mean of 20°.9 and 38°.9 and the mean of 0.1494 and 0.1485, we have, at 29°.9, $K = 0.1490$. Taking this as a starting point, we find, for the temperature coefficient of K between 29°.9 and 56°.2,

$$\frac{0.1490 - 0.1453}{0.1490\,(56.2 - 29.9)} = -0.00094.$$

Similarly, we find between 29°.9 and 74°.7,

$$\frac{0.1490 - 0.1447}{0.1490\,(74.7 - 29.9)} = -0.00064.$$

So much for the May and June numbers.

From the July and August numbers we get, between 27°.6 and 56°.8,

$$\frac{0.1505 - 0.1458}{0.1505\,(56.8 - 27.6)} = -0.000107.$$

The mean of all these estimates of the temperature coefficient of K is —0.00075, according to which the thermal conductivity of the cast iron disk diminishes about 1% for each 13°.3 rise of temperature within the limits of the observations above recorded. According to experiments described in Appendix II. following this paper, the temperature coefficient of electrical conductivity of the same cast iron, between 17° and 67°, is —0.00118; which means that the electrical conductivity between these limits diminishes at the rate of 1% for each 8.°5 rise of temperature. At one time during the preparation of this paper it appeared that the two temperature coefficients were very nearly equal. This led to a more careful examination of the evidence than had been made before, and a repetition of certain measurements, with the result given above. It may yet be that the two coefficients are equal. Where both are so small the question of equality or inequality is difficult to settle, although

a new series of experiments with same cast iron disk would probably give results much more concordant than those set down in this paper. In wrought iron the temperature coefficient of electrical conductivity is much greater than in cast iron, and if the temperature coefficient of thermal conductivity is correspondingly large in wrought iron, fairly accurate measurements of this latter coefficient should be attainable with this material. A disk of wrought iron will probably be put to the test before long. The disk of mild steel used in the experiments described in a preceding paper was very like wrought iron in many respects; but it has already been stated, in the first part of this paper, that the experiments with this disk were not entirely satisfactory, the disk itself and its copper coverings being too thin for the best effect.

The experiments of this paper have given a larger value of K, for the piece of cast iron dealt with, than was expected. It is much larger than the value, about 0.105, found some years ago for two specimens of cast iron near 115°C. by one of the authors * of this paper, using the method of Forbes. It is much larger than the values found by Kohlrausch and by Wiedemann and Franz for soft steel near 15°. Nevertheless, there seems to be no good reason for doubting the substantial accuracy of the value of K found in this paper. The most novel, and perhaps the most doubtful, feature of the method here described is the use of the iron itself as part of a thermo-electric element. How carefully the thermo-electric behavior of the iron with respect to copper has been considered will be apparent to the reader of Appendix I.

Another subject of possible doubt is the amount of error caused by neglect of radiation or convection between the water jacket, Figure 2, and the apparatus surrounded by it. The value found for K is affected, 1st, by such interaction as occurs between the jacket and the disk; 2d, by that between the jacket and those surfaces which lie above the disk and below J_1 and J_2. The mean temperature of the curved surface of the disk was probably four or five degrees below the temperature of the jacket when the warm stream ran above, and nearly an equal amount above that of the disk when the cold stream ran above. The area of this surface between the two hard rubber rings $h\,h$ and $h'h'$ was about 50 sq. cm. Preston, "Theory of Heat," p. 461, gives, as found by McFarlane for a blackened sphere suspended within a water jacket 5° cooler than itself, "*heat emitted per second, per degree difference of temperature,*

* E. H. Hall, in these Proceedings, 1892, p. 262.

per square centimeter in water-gram units" equals 0.000252. Assuming this rate of emission or absorption for each of the 50 sq. cm. of the curved surface of the disk, we should get for the passage of heat per second $50 \times 0.000252 \times 5 = 0.063$ units. The heat passing per second through the disk from face to face was usually about one hundred times as much as this. If we assume, for the moment, that the passage of heat through the curved surface of the disk is equal to 1% of that which flows from face to face, we may thereupon reason as follows. When the warmer stream flows above the disk, the disk takes in heat from the jacket, and the total amount passing out through the lower face of the disk exceeds by 1% the amount flowing in at the upper face. The inflow at the curved surface distorts the isothermals and lines of flow within the disk in such a way that, with a given difference of temperature between the faces, the flow from face to face, which is equal to the inflow at the upper face, is less than it would be if the flow within the disk were *adiabatic*, that is, if there were no inflow at the curved surface. On the other hand, the outflow at the lower face is greater than the adiabatic flow from face to face would be. We may conclude that, under the conditions assumed, the actual inflow at the upper face is about 0.5% less than the adiabatic flow would be with like temperatures at the faces, and that the outflow at the lower face is about 0.5% greater than the adiabatic flow would be. Our method of calculating K assumes adiabatic flow, while our observations give us the inflow at the upper surface. Accordingly, under the conditions here assumed, the value obtained for K would be about 0.5% too small. A similar error, *in the same direction*, would be made with the colder stream flowing above the disk, so that we could not eliminate it by combining two sets of observations. In fact, however, the assumption that the rate of transmission at each square centimeter of the curved surface of the disk, thickly wrapped with cotton, is the same as that found by McFarlane for a bare blackened surface of copper, gives a very large overestimate of the possible error from this source.

Turning now to the surfaces between the top of the disk and the parts J_1, J_2, Figure 2, we find the area of these surfaces to be about 300 sq. cm. No systematic observations of the temperature of the jacket were kept during the experiments of which this paper gives an account, but from previous observations it appears probable that the mean temperature of the jacket was about 1° C. lower than the temperature of the parts which we are now considering, when the warmer stream ran above, and 0°.5 or less higher than the temperature of these parts when the colder stream ran above. Assuming the difference of temperature

to have been 1° C., and assuming, for the moment, the same rate of surface transmission which we have used above, we get, as the amount of heat passing per second between the parts considered and the jacket, $300 \times 0.000252 = 0.0756$ units. This is, perhaps, rather more than 1% of the heat carried from face to face through the disk, and if it were a fair estimate of the actual transmission between the jacket and the surfaces considered, the neglect of this transmission would make K, as calculated, about 1% too large. This error would not be eliminated by combining sets of observations, some with the warmer stream above, and some with the colder stream above. But in this estimate, as in that relating to the action at the curved surface of the disk, the rate of transmission assumed is no doubt much too large, the surfaces enclosed by the jacket being, for the most part, well wrapped with cotton.

It seems, therefore, unlikely that any considerable error was made by neglecting the interchange of heat between the water jacket and the apparatus within it.

There is little doubt that much more concordant values of K than those given in this paper can be obtained by a somewhat more careful control of the temperature of the water, and by making each set of observations longer than the sets, often very brief, which were made in the investigation which has here been described.

Summary.

The thermal conductivity of the cast iron used is about 0.1490 at 30° C. The temperature coefficient of thermal conductivity, if Winkelmann's rule for the change of specific heat of water with temperature is correct, appears to be about −0.00075 between 20° and 75°, so that a rise of about 13° C. corresponds to a fall of 1% in conductivity.

If the change of specific heat of water between 30° and the higher temperatures up to 75° were neglected, the value found for the temperature coefficient would be about −0.0010.

The electric conductivity of this cast iron is about 112,200 in c. g. s. units. (See Appendix II.)

The temperature coefficient of its electric conductivity between 17° and 67° is about −0.00118.

The method used appears to be capable of giving better results than have yet been obtained by it.

APPENDIX I.

Measurement of the Thermo-Electric Quality of Short Iron Bars.

When it became necessary to determine, relatively to copper, the thermo-electric quality of the cast iron disk thickness-wise, the problem appeared to be one of some difficulty. The thickness of the disk was about 1.8 cm. The thickness of the slab from which the disk had been taken was such that bars 2 cm. long could be cut from it thickness-wise; but to make satisfactory thermo-electric measurements upon a single bar of this length appeared to be impracticable. The device of putting a number of such bars end to end, so as to make a column of considerable length, and placing this column lengthwise between two blocks of copper of different temperatures, seemed a hopeful one; but it had to be put to the proof before it could be used with confidence.

Accordingly a very soft magnet core rod, about 0.16 cm. in diameter, was taken, and from it were cut one piece 15 cm. long and ten pieces each 2 cm. long. Copper wires were soldered to the ends of the 15 cm. piece, and this piece was then mounted very much as the piece $I_1 I_2$ is mounted in Figure 4. The parts exposed to the streams were now, however, some 5 cm. long, about twice as long as the exposed parts in similar preceding tests. A thin coating of paraffine was now used to protect these parts from the chemical action of the water.

The ten 2 cm. pieces, after being carefully cleaned and polished at the end surfaces, were placed end to end in a wooden tube, which in all its dimensions was much like the wood of a common pencil from which the graphite has been taken out. The iron, corresponding in position to the graphite of a pencil, projected from the wood about 0.2 cm. at each end. $R R$ in Figure 5 (Plate II) shows in diagonal lines a section of the wooden tube, or rod, the iron within being indicated by a heavy black line; the scale of the figure is $\frac{1}{2}$. Water jackets, J_1 and J_2 in Figure 5, surrounded $R R$ for the greater part of its length. The iron column projecting from $R R$ was pressed between two copper blocks $B_1 B_1$ and $B_2 B_2$, through which flowed streams of water at any temperature required. The pressure was applied by means of a wooden plunger p, supported in the block b, and pushed against $B_2 B_2$ by a fairly constant force. The blocks $B_1 B_1$ and $B_2 B_2$ were of the same diameter as the jackets j_1 and j_2, and all of these objects rested in a slot cut lengthwise in a piece

of hard wood. S, S, S, in the figure, are parts of this wooden support which would have been cut through by a vertical longitudinal section through the middle of the apparatus. Certain edges of the support which do not form part of the section shown are indicated in the figure by light dotted lines. Behind, to the left of, the block B_1 B_1, is shown a copper wire W_1, about 0.1 cm. in diameter, which extends through the centre of a wooden rod r and bears against B_1 B_1, thus making a back-stop for the pressure exerted at the other end of the apparatus. From the block B_2 B_2 another copper wire, W_2, held in firm contact with B_2 B_2, leads away. The wires W_1 and W_2 are parts of the thermo-electric circuit of the apparatus, and are in metallic connection with the terminals of a galvanometer. B_1 B_1 is provided with a water jacket J_1 J_1, the construction of which is indicated by certain lines in Figure 5 and in Figures 6, 7, and 8. Thus, Figure 6 shows a vertical cross-section through J_1 J_1 near the left end of B_1 B_1, the dotted lines indicating certain edges not lying in this section. Figure 7 shows a vertical cross-section through J_1 J_1, through B_1 B_1, and through the thermometer T_1 (Fig. 5). Figure 8 shows a horizontal section through J_1 J_1 and B_1 B_1. The block B_2 B_2 is protected by a jacket quite similar to J_1 J_1. Wads of cotton were used to protect certain parts of each block which were not covered by the jackets.

The course of the water through the apparatus is indicated by arrows. Thus, at the left hand the streams enters at A, passes down along the bulb of T_1 through B_1 B_1, thence by a rubber tube, longer in fact than in the figure, to J_1 J_1, thence by another rubber tube to j_1, and out at E_1. The flow of the right hand stream is strictly analogous. Each stream usually carried 20 or more grams of water per second. The thermometers T_1 and T_2 were the same that were used with the 15 cm. bar of iron and in previous tests of thermo-electric junctions. Sets of observations at a given mean temperature were made in pairs, one set having T_1 the warmer, the other set having T_2 the warmer.

It was necessary to give careful attention to the electrical resistance of the column of short iron bars; for it could not be safely assumed that this resistance would be either small or constant. It was found, naturally, to depend somewhat upon the magnitude of the pressure applied at the ends of the column. In the experiments upon soft iron which we are just now considering, the pressure was exerted by means of a compressed piece of india-rubber tubing, not shown in Figure 5. In later experiments, with cast iron, it was applied through a lever as in Figure 5, the force F being exerted upon the end of the lever by means of a

spring balance at a point too far down to be shown in place. With the rubber tube in use the pressure against the end of the column was perhaps 1.8 kilograms. When the lever and balance were used, it was sometimes about three kilograms and sometimes less.

The various jacketing and protecting devices shown in Figure 5 were not all used at first, and in the early experiments on soft iron the e. m. f. obtained from the column of short bars was several per cent less than that obtained from the 15 cm. soldered bar, with a given difference of temperature between the two thermometers. This discrepancy gradually diminished as the method of experimentation was improved, until at last it became little or nothing, as the following numbers, obtained with the system of jacketing shown in Figure 5, will testify.

	With Soldered Bar.		With Column of Short Bars.	
Date, 1898.	Mean Temp.	E. M. F. Per Degree Diff. of Temp.	Mean Temp.	E. M. F. Per Degree Diff. of Temp.
April 9	14°.5 C.	[214.7]	14°.5	[209.6]
" "	18°.4	214.2	18°.4	212.6
" 20	20°.9	213.2	20°.9	211.0
" "	16°.9	215.8	17°.0	214.3
" 22	18°.4	214.2	18°.4	214.4

The e. m. f. is here given in terms of a purely arbitrary unit. The values in brackets were obtained under conditions of special uncertainty as to resistance. Considering the final trials of the end-to-end short bar method satisfactory, I proceeded to apply it to cast iron. From the end of the slab that furnished the conductivity disk a slice was cut crosswise, about 10 cm. long, 2.5 cm. wide, and 0.3 cm. thick. This was cut up into 26 parts, and each of these parts was turned down to a thickness of about 0.16 cm.; or, rather, 18 of them were so treated, the other 8 being broken at some stage of the operation. They were then boiled for about 20 minutes in a strong solution of caustic potash, partly to free them from oil, partly because the disk had been thus heated before its conductivity was tested. In all of this work an attempt was made, and I think a successful one, to keep the bars in the same order with respect to each other that they had before being cut from the slice, so that I could at the end tell what bars had been taken from near the end of the slice and what ones from near the middle.

When ready for the test of thermo-electric quality, I rubbed the flat ends of each little bar bright with infusorial earth, and wiped them carefully; for it is evident that a particle of dirt or of vegetable fibre left

upon one of the ends may break altogether the electric circuit of which the column of bars should form a part. In later work it seemed better to rub the ends with fine emery paper, and then wipe them upon smooth hard-finished paper to remove adhering particles of dust.

Bars 1, 3, 5, 7, 9, 11, 14, 17, 19, and 23, the numbers indicating their order from one end toward the other of the slice from which they had been cut, were placed in the order just given, end to end in the apparatus shown by Figure 5. The resistance of the column, which in the case of soft iron bars had been about 2 ohms under a pressure of 1.8 kgm., was now found to be surprisingly large. It diminished with increase of pressure, but even with a pressure of 3 kgm. was at first, June 30, about 16.5 ohms. Under a nearly continuous application of this pressure it gradually grew less, until, on July 2, it was about 5 ohms, after which it changed but little, although it appeared to be somewhat greater on July 4.

With this set of bars, and with the method already described, the following results were obtained : —

Date.	Diff. of Temp. between Ends.	Mean Temp.		E. m. f. in Volts per Degree.	
June 30, 1898,	9°.84	29°.3	29°.7	.00000549	.00000549
July 4, "	10°.21	30°.1		.00000549	
" " "	11°.97	45°.2			.00000593
" 2, "	13°.40	63°.2	63°.1	.00000649	.00000646
" 4, "	15°.13	63°.0		.00000643	

On November 4, 1898, observations were made in the same way with ten cast iron bars taken from the same set as those used in June and July; whether the same bars or not, could not be told. The result was :

Date.	Diff. of Temp.	Mean Temp.	E. m. f. per Degree.
Nov. 4	10°.24	29°.7	.00000550

It should be remembered that the end to end method of experimentation with short bars of cast iron was adopted because of a doubt as to the availability of the thermo-electric test made by a different method on a 10 cm. bar cut crosswise from the cast iron slab. This earlier method had given : —

Mean Temp.	E. m. f. per Degree.
14°	.00000507
18°.6	.00000518
40°.6	.00000576
63°.3	.00000647

A comparison of these results with those obtained by the end to end method with short bars, cut thickness-wise from the slab, shows that the two methods gave almost identical results. Of course, it is possible that the bars used in the two methods differed considerably in thermo-electric quality, and that some error in one or the other method compensated for and obscured this difference of quality; but it is much more reasonable to conclude that the slab from which all of the bars were cut had practically the same thermo-electric quality crosswise as thickness-wise, and that the accuracy of each method of testing this quality is affirmed by its concordance in results with the other method. The results of both methods were used for plotting a line from which values of the copper-iron thermo e. m. f. could be derived for purposes of interpolation. This line is a curve ascending with increase of temperature, and slightly concave upward. The divergence of this line from true rectitude is probably not very significant. There is in the corresponding curve for the thermo e. m. f. of the copper-German silver junctions, described in the preceding pages, a divergence of about the same relative amount in the same direction. It is possible that this peculiarity of both lines is due to some idiosyncrasy of the thermometers used in the thermo-electric tests. The same thermometers were used in all these tests; and therefore, as the method of calculation of conductivity involved the ratio of the e. m. f. of copper-iron and copper-German silver, no final error as to conductivity results from any small imperfections of these thermometers.

E. H. H.

APPENDIX II.

Measurement of Electric Conductivity of the Cast Iron.

One of the 2 cm. bars described in Appendix I. was used for this determination. Four copper wires were attached to this bar by electrolytic deposit of copper. Two of the wires were about 0.08 cm. in diameter; these were attached to the flat ends of the bar, and served to carry in and out an electric current of about 0.25 ampere. The other two wires were much finer, about 0.018 cm. in diameter; these were attached at two points about 1.7 cm. apart, each being about 0.15 cm. from one end of the bar, and were used for making connection with a potentiometer. The bar was submerged in oil during the measurements. The temperature of the oil was controlled by water flowing through

a lead tube bent into solenoidal form. The bar was placed horizontal within the solenoid, the axis of which was vertical.

The electrical resistance in absolute c. g. s. measure was about 112,200 at 17°.4.

From the observation of September 20, 1898, the temperature-coefficient of conductivity between 20°.9 and 61°.2 appeared to be −0.00120. The observations of September 27, between 17°.4 and 67°.4, gave −0.00116. We may take the mean, −0.00118. In both cases the coefficient was calculated by the formula

$$\textit{Coefficient} = \frac{\text{Cond. at high temp.} - \text{cond. at low temp.}}{\text{Cond. at low temp.} \times (\text{high temp.} - \text{low temp.})},$$

without reference to 0°.

E. H. H.

Fig. 3

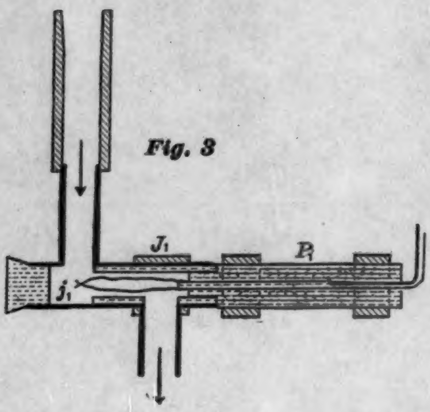


Fig. 1

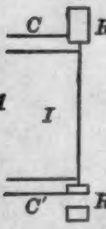
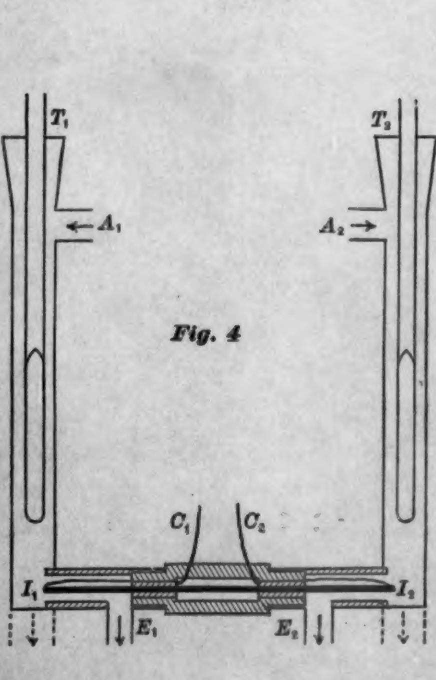


Fig. 4



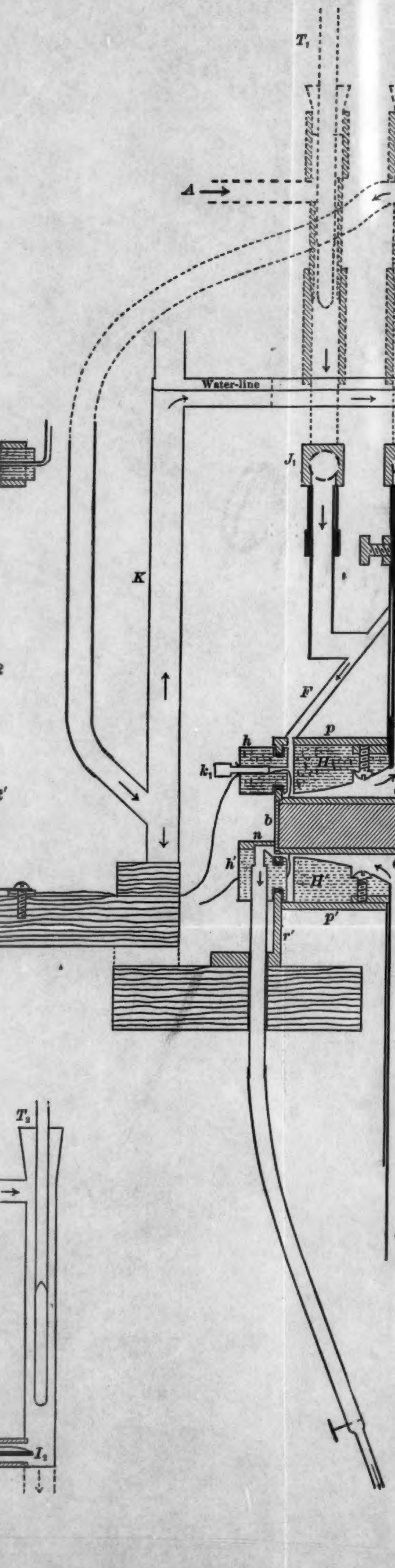


PLATE I.

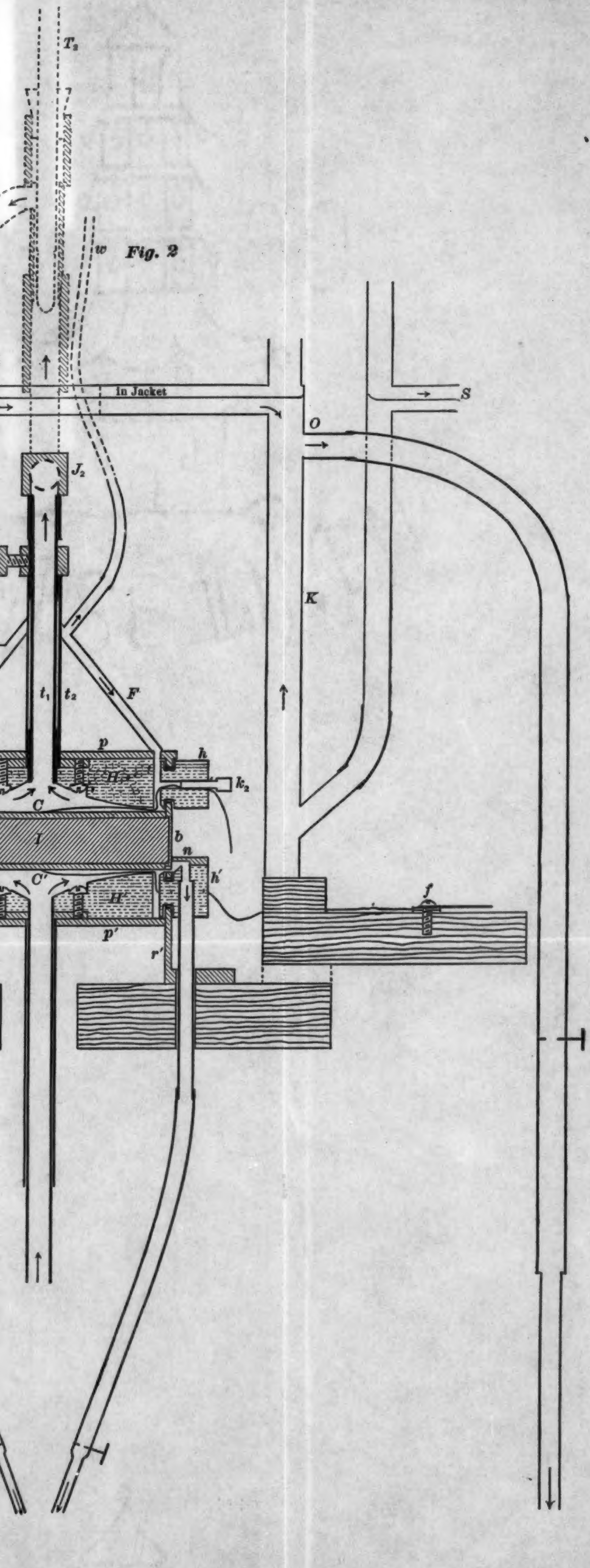


Fig. 6

Fig. 7

Fig

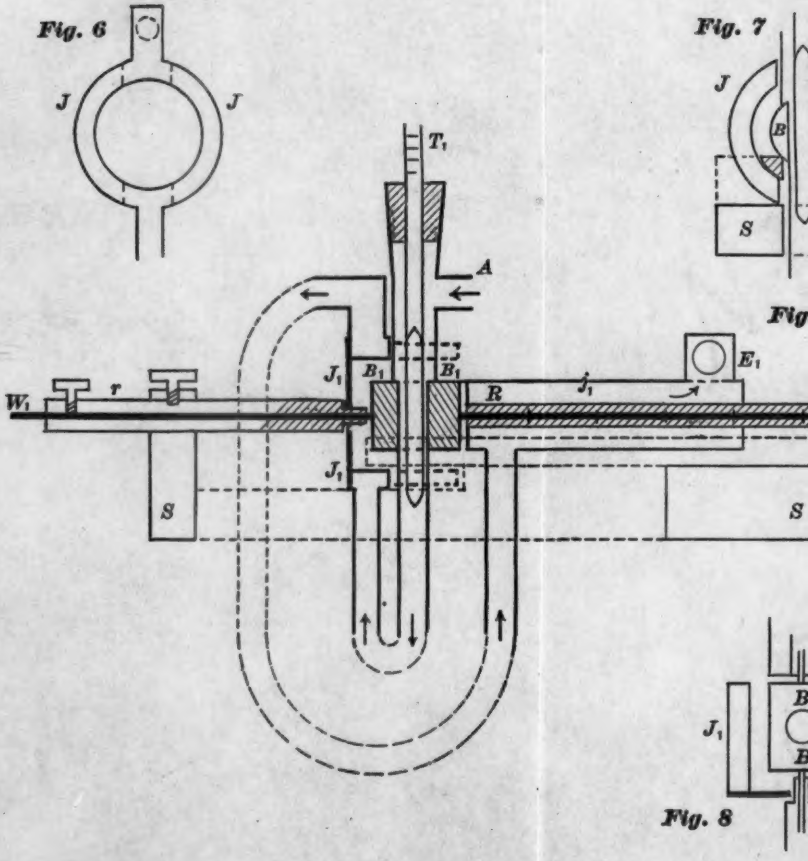


Fig. 8

PLATE II.

Fig. 5